University of California, Berkeley
Physics 110B Spring 1997 (*Strovink*)

MIDTERM EXAMINATION

Directions. Do all three problems, which have unequal weight. This is a closed-book closed-note exam except for one $8\frac{1}{2} \times 11$ inch sheet containing any information you wish on both sides. Calculators are not needed; where numerical results are requested, 30% accuracy is sufficient. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (35 points) A coaxial transmission line consists of two perfectly conducting circular cylindrical thin-walled tubes of radii a and b, respectively, both centered on the $\hat{\mathbf{z}}$ axis. The region $a < r < b$ is evacuated. Consider propagation of electromagnetic waves in the TEM mode ($E_z = B_z = 0$) within the vacuum region. Take

$$\mathbf{E}(\mathbf{r},t) = \Re(\tilde{\mathbf{E}}(r,\phi)e^{i(kz-\omega t)})$$
$$\mathbf{B}(\mathbf{r},t) = \Re(\tilde{\mathbf{B}}(r,\phi)e^{i(kz-\omega t)}) ,$$

where $k = \omega/c$. Then, in the vacuum region, Maxwell's equations reduce to

$$\nabla_t \cdot \tilde{\mathbf{E}} = \nabla_t \times \tilde{\mathbf{E}} = 0 ,$$

and

$$c\tilde{\mathbf{B}} = \hat{\mathbf{z}} \times \tilde{\mathbf{E}} ,$$

where

$$\nabla_t \equiv \nabla - \hat{\mathbf{z}}\frac{\partial}{\partial z} .$$

(a) (5 points) Show that $\tilde{\mathbf{E}}$ can be written as

$$\tilde{\mathbf{E}}(r,\phi) = -\nabla_t \tilde{\Phi}(r,\phi)$$

where

$$\nabla_t^2 \tilde{\Phi} = 0 .$$

(If you don't manage to show this, you may nevertheless assume this result in the later parts.)

(b) (15 points) Assume that $\tilde{\Phi} = \Phi_0$ on the outer cylinder, and $\tilde{\Phi} = 0$ on the inner cylinder, where Φ_0 is a real constant. Calculate the physical (real) fields $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ in the gap between the cylinders, in terms of Φ_0.

(c) (15 points) Find Z_0, the characteristic impedance of this transmission line. Z_0 may be defined as the ratio of Φ_0 to the maximum total current flowing on either cylindrical surface. Assume that this current is distributed uniformly in ϕ. Evaluate Z_0 in ohms for the case $b/a = 2.71828$.

Problem 2. (30 points) Event A happens at spacetime point $(ct,x,y,z) = (2,0,0,0)$; event B occurs at (0,1,1,1), both in an inertial system S.

(a) (10 points) Is there an inertial system S' in which events A and B occur at the same spatial coordinates? If so, find $c|t'_A - t'_B|$, c times the magnitude of the time interval in S' between the two events.

(b) (10 points) Is there an inertial system S'' in which events A and B occur simultaneously? If so, find $|\mathbf{r}''_A - \mathbf{r}''_B|$, the distance in S'' between the two events.

(c) (10 points) Can event A be the cause of event B, or vice versa? Explain.

Problem 3. (35 points) A point charge e travelling on the x axis has position

$$\begin{aligned}\mathbf{r}(t) &= +\hat{\mathbf{x}}\frac{ct}{2} \quad (t<0)\\ &= -\hat{\mathbf{x}}\frac{ct}{2} \quad (t>0)\ .\end{aligned}$$

That is, the charge reverses direction instantaneously at $t=0$, while it is at the origin. The fields that the charge produces are viewed by an observer at $(x,y,z)=(0,1,0)$ m.

(a) (20 points) What magnetic field $\mathbf{B}$ does the observer see at $t=0$?

(b) (15 points) At time t such that $ct=1$ m, what is the direction of the electric field $\mathbf{E}$ seen by the observer? (You need consider only the part of the total electric field which is dominant at exactly that time.) Justify your answer.